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Research

Location of Points on Plane and the Order of Disposition of Sum of Powers of Cardinal Coordinates

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Abstract
This article exposed the structure of all orders of the sum of powers of cardinal coordinates with the explicitly determined computational disposition of similarity, between the coefficient of binomial expansion and the sum of powers of pairs of cardinal points on the plane. The proofs were achieved by a logical deployment of combinatorial techniques, laws of indices on powers of cardinal bases, and a comparison of corresponding results. The results proved conclusively that the sum of the powers of cardinal points is equal to the coefficients of the Binomial expansion with respect to the Pascal triangle pattern and entries.

Theorem, the sum of powers of cardinal coordinate equals the corresponding elements of Binomial coefficient at every stage on a plane $\int_n (x + y) = {}^nCr : n \geq 0, r \leq n, r \epsilon [0, n] \not< n \geq 0$.

1.0 Introduction

Cardinal points are the four main points; North, East, South and West (N, E, S, W) of the compass. A compass is an instrument for finding direction with a needle that always point to the north. The compass rose also called the wind rose, contained four or more pointed figures that can be seen on a compass, map, nautical chart, or movement. The four points (N, E, S, W) are known as the primary directions and the intermediate directions that lie between the four cardinal directions. These intermediate directions are also known as the intercardinal or ordinal directions. The primary intercardinal directions are NE, SE, SW and NW. The directions are further broken down into secondary intercardinal directions. They are rarely used. They are applicable in survey control, measuring angles, a global positioning system (GPS), satellite ranging, aeronautical science and navigational science, and also on the globe in general to locate points. They are used to identify polar axis on the globe. The cardinal points are used in finding directions and inclinations known as bearings in mathematical calculations. Thus, they define points in space and on the earth. Every point on the circumference of a circular region can be defined with respect to cardinal points. In astronomy, planetary bodies' motion is defined with cardinal points. The coordinates used by astronomers are similar to that used by navigators in the ocean or sea. The grids consist of one set of lines running East to West on the celestial equator and another running North to South connecting one celestial pole to the other.

The East-West lines play the same role as latitude on the Earth, but to avoid confusion with terrestrial coordinates, they are called lines of declination. The North-South lines play the same role as the longitude of ascension. For example, a celestial object may be located by its altitude and azimuth. In this horizon system, altitude is an object's angle above the horizon.

Azimuth is generally defined as the angle measured Eastward along the horizon from North to the point below the object although sometimes it is measured from South, whichever convention is used for azimuth, these coordinates are useful for pointing out objects or tracking their motion.

This paper presents a computational examination of the cardinal points (North, East) of the first quadrant pair of cardinal perpendicular (at 90^0). The pair of cardinal points termed cardinal coordinates is derived from the product of any two neighboring coordinates (pair of cardinal points) with respect to the angle of inclination between the two neighboring points.

The result reveals that if we assume that the cardinal points (N, E) are points with powers (*x*, *y*), then $(N^x, E^y) = (P^x P^y)$ where N $\Rightarrow$ Point $\Rightarrow$ E

$\Rightarrow$ N = E = Point = P.

$\Rightarrow (N^x, E^y) = (P^x P^y) = P^{x+y} = (NE) = (NE)^{x+y}$

The sum of $x + y$ corresponds to an entry of the Pascal triangle, inclined at a given angle 0^0 on the perpendicular plane. The emerging pattern motivates the investigation of whether or not this pattern will persist for all the coefficients of the binomial theorem for all ($n \geq 0$) positive sets of integers.

Cardinal points of an astronomical body as seen in the sky are four points, defined by the directions toward which the celestial poles lie relative to the center of the disk of the in the sky, Meadows and Peter. (2013).

A study of co-ordinate-wise-powers of subvarieties of p^n i.e. varieties arising from raising of all points in a given subvariety of p^n to the r-th power coordinate by coordinate, which corresponds to studying the image of a subvariety of p^n under the quotient of p^n by the action of the finite group Z^{n+1}. Applying to compute the degree of the variety of ortho stochastic matrices and determine iterated dual and reciprocal varieties of powers sum hyper-surfaces, Papri et. al. (2020). A work that describes how to build a model of the Pascal pyramid displaying the arrangement in the space of trinomial coefficients, Hans et. al. Hans (1975).

A research on cardinal points provide direct evidence demonstrating that daily experience with the cardinal coordinates system i.e. (east, west, north and south). Xin Hao et. al. (2006), the ability to coordinate system play a vital role to connect scattered locations into cognitive map.

A research work shows that over a given finite field, a method of obtaining explicit expressions for satisfying certain conditions. As illustrations, they present a series of new formulas and obtain simple proofs known as formals, Dan and Anders (1994).

The result obtained in the first quadrant North to East is similar to results in each of the other quadrant with same pattern. Therefore, the examination was restricted on the first quadrant North to East at an angle of (90^0). The sum of powers applied in the plane was studied by Pedro and Vilhena, (2003). In their work, they propose a demonstration that if the sum of powers where A^x and B^x have a common prime factor and generates a compound number C^z, then Beal's conjecture involving.

$A^x + B^y = C^z$ is true for all A, B, C, x, y, z positive integers and x, y, z > 2 as well as showing types where the sum $A^x + B^y$ generates no prime factor among A, B, and C. In 1993, Andrew Beal began to investigate new hypothesis by Fermat's theorem of which among one of them was the idea of sum of powers. In their contribution in the Diophantine Equations with power sums, M. Lauret (2017) applied the subspace theorem to study solutions $X = (X_1, \ldots\ldots, X_r) \in Z^r$ of polynomial exponential equations.

$$\sum_{i=1}^{N} P_i(x) a_i^x = 0$$

Carjava and Zannier [7] studied these and more general equations in his article in which power sum are in the form: $U(n) = b_1 a_1^n + \ldots\ldots\ldots + b_m a_m^n$. where $a_1, \ldots. a_m$ (the roots) and $b_1 \ldots..,\ b_m$ (the coefficients) are complex numbers. The main result of Corvaja and Zannier concerns a Q-power sum with positive roots, where Q is any field characteristics.

More generally, in the sequel, the question at the heart of the matter is the following: what is the computational disposition of orders of sum of pairs of cardinal coordinates on a first quadrant of the Cartesian plane (Rene Descarte). Review of literature shows that no such investigation has been undertaken. Thus, this article adds to the existing body of knowledge, by providing answers to the above question.

2.0 Materials and Methods

2.1 Preliminary Definitions

In what follows, the cardinal coordinates, sum of powers, range of angle 0^0 will be defined.

2.1.1 Cardinal Coordinates

North = N $\Rightarrow$ Point/Path = P_n

East = E $\Rightarrow$ Point/Path = P_n

$\Rightarrow$ (North, East) = (N, E) = P_n where $n \geq 0$.

If N = $90^0 \Rightarrow$ E = 0^0 (anti-clockwise).

If N = $0^0 \Rightarrow$ E = 90^0 (clockwise).

$$0^0 \leq \theta^0 \leq 90^0$$

2.1.2 Sum of Powers of Cardinal Coordinates

$\int_n = (N^x, E^y) = P_n^x . P_n^y = P_n^{x+y} = x + y = {}^nC_r$

$n \geq 0, r \leq n$.

2.1.3 Corollary

$$\int_n = (N^x E^y) = P_n^{x+y} = \begin{cases} equal\ 45^0, if\ x = y \\ \qquad\qquad \not\prec n \geq 1 \\ not\ equal\ 45^0, if\ x \neq y. \end{cases}$$

$n \epsilon [1, \infty]$

2.2 EXPERIMENTATION AND PROOFS

The figure below (Fig. 1) represents the disposition of cardinal coordinates with their respective powers. The vertical line represents the North direction while the horizontal line represents the East direction. The powers of the cardinal points are the frequency of the direction which indicates the breakdown of the primary intermediate and the secondary ordinal directions. The cardinal points are represented in pairs where $N=N^1E^0$ and $E=N^0E^1$.

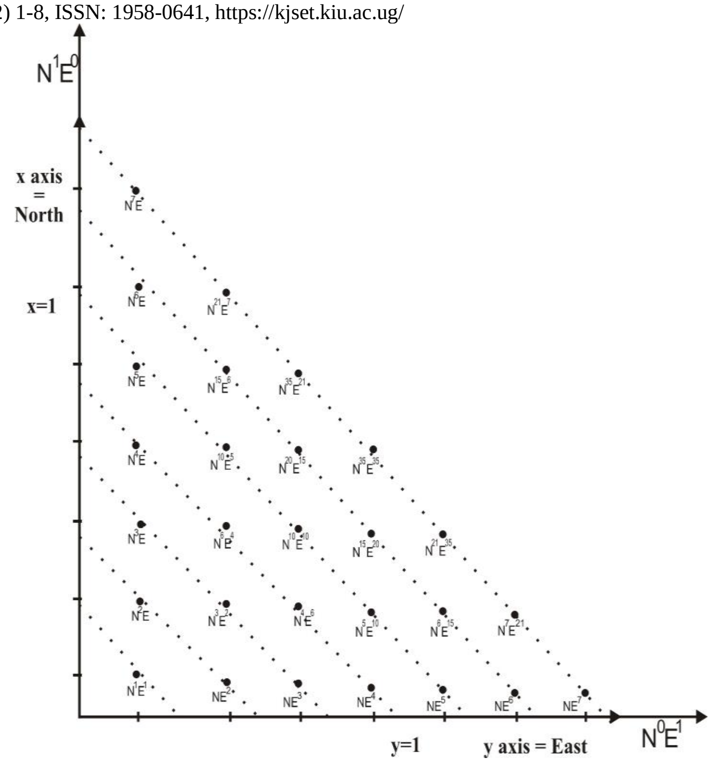

Fig. 1: Disposition of cardinal coordinates on the plane

Fig. 2 represents the pairs of points in form of the powers of the intermediate coordinates known as coordinate points in the plane, where North is positive x axis and East is the positive y axis. The ordinal direction are represented as powers which implies that:

$N^xE^y = (x, y)$

Where (x, y) are pairs of a location on the plane, representing powers of cardinal direction.

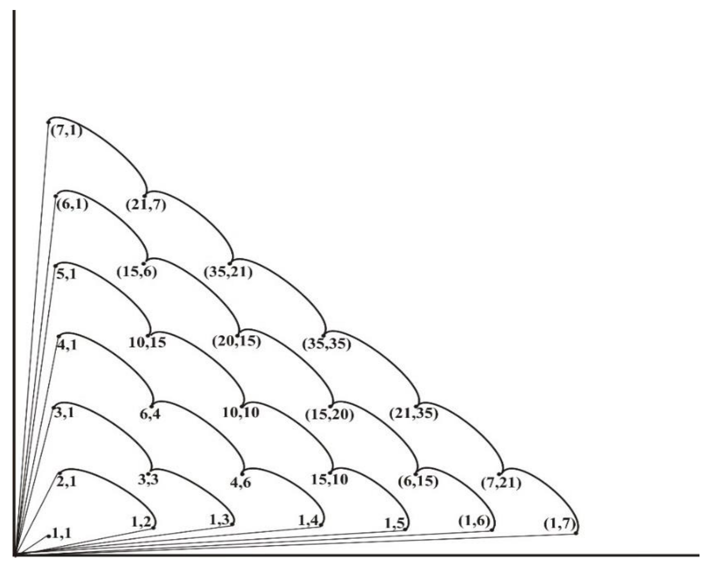

Fig. 2: Pairs of powers of cardinal coordinates

The values in the Fig. 3 are the sum of powers of the cardinal directions $N^xE^y = (x, y)$, such that the sum X = Y is the value at each stage. These values correspond to the elements of the entries of Pascal triangle.

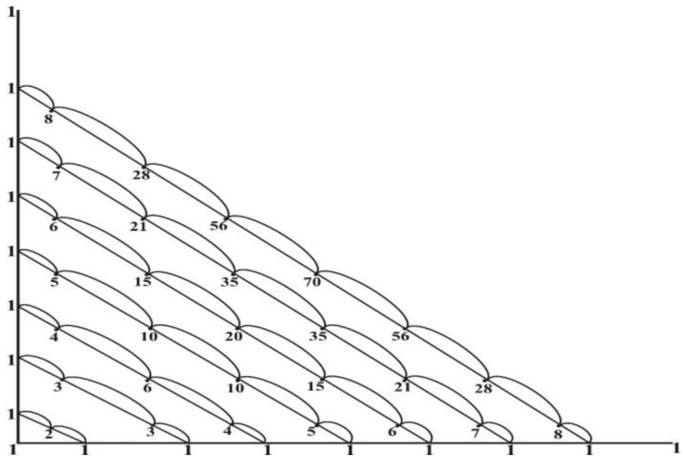

Fig. 3: Value of sum of powers of cardinal coordinates.

The triangular pattern shown in Fig. 4 is similar to the triangular pattern of Pascal triangle. The cardinal coordinates with their respective powers form the pattern to the 9th row.

$$\int_0 = NE^0$$
$$\int_1 = NE^0 \sim N^0E$$
$$\int_2 = NE^0 \sim NE \sim N^0E$$
$$\int_3 = NE^0 \sim N^2E \sim NE^2 \sim N^0E$$
$$\int_4 = NE^0 \sim N^3E \sim N^3E^3 \sim NE^3 \sim N^0E$$
$$\int_5 = NE^0 \sim N^4E \sim N^6E^4 \sim N^4E^6 \sim NE^4 \sim N^0E$$
$$\int_6 = NE^0 \sim N^5E \sim N^{10}E^5 \sim N^{10}E^{10} \sim N^5E^{10} \sim NE^5 \sim N^0E$$
$$\int_7 = NE^0 \sim N^6E \sim N^{15}E^6 \sim N^{20}E^{15} \sim N^{15}E^{20} \sim N^6E^{15} \sim NE^6 \sim N^0E$$
$$\int_8 = NE^0 \sim N^7E \sim N^{21}E^7 \sim N^{35}E^{21} \sim N^{35}E^{35} \sim N^{21}E^{35} \sim N^7E^{21} \sim NE^7 \sim N^0E$$
$$\int_9 = NE^0 \sim N^8E \sim N^{28}E^8 \sim N^{56}E^{28} \sim N^{70}E^{56} \sim N^{56}E^{70} \sim N^{28}E^{56} \sim N^8E^{28} \sim NE^8 \sim N^0E$$

Fig. 4: Triangular form of cardinal coordinates

The Fig. 5 represents the pairs of powers of cardinal coordinates in a triangular form. The pairs of points are sum up to a value which correspond to the values in the triangular array of binomial coefficients called Pascal triangle.

$$\int_0 = (1,0)$$
$$\int_1 = (1,0) \sim (0,1)$$
$$\int_2 = (1,0) \sim (1,1) \sim (0,1)$$
$$\int_3 = (1,0) \sim (2,1) \sim (1,2) \sim (0,1)$$
$$\int_4 = (1,0) \sim (3,1) \sim (3,3) \sim (1,3) \sim (0,1)$$
$$\int_5 = (1,0) \sim (4,1) \sim (6,4) \sim (4,6) \sim (1,4) \sim (0,1)$$
$$\int_6 = (1,0) \sim (5,1) \sim (10,5) \sim (10,10) \sim (5,10) \sim (1,5) \sim (0,1)$$
$$\int_7 = (1,0) \sim (6,1) \sim (15,6) \sim (20,15) \sim (15,20) \sim (6,15) \sim (1,6) \sim (0,1)$$
$$\int_8 = (1,0) \sim (7,1) \sim (21,7) \sim (35,21) \sim (35,35) \sim (21,35) \sim (7,21) \sim (1,7) \sim (0,1)$$
$$\int_9 = (1,0) \sim (8,1) \sim (28,8) \sim (56,28) \sim (70,56) \sim (56,70) \sim (28,56) \sim (8,26) \sim (1,8) \sim (0,1)$$

Fig. 5: Triangular form of powers of cardinal points in pairs

The powers of the cardinal points are represented in Fig. 6 using the combinatorial form which are the binomial coefficients where each value can be expressed in the form: $\left(\frac{k}{i}\right)$

This shows that the sum of powers of the cardinal points are the binomial coefficients.

$$\int_0 = N^{\binom{0}{0}}$$
$$\int_1 = N^{\binom{1}{0}} \sim E^{\binom{1}{1}}$$
$$\int_2 = N^{\binom{2}{0}} \sim (NE)^{\binom{2}{1}} \sim E^{\binom{2}{2}}$$
$$\int_3 = N^{\binom{3}{0}} \sim (NE)^{\binom{3}{1}} \sim (NE)^{\binom{3}{2}} \sim E^{\binom{3}{3}}$$
$$\int_4 = N^{\binom{4}{0}} \sim (NE)^{\binom{4}{1}} \sim (NE)^{\binom{4}{2}} \sim (NE)^{\binom{4}{4}} \sim E^{\binom{4}{4}}$$
$$\int_5 = N^{\binom{5}{0}} \sim (NE)^{\binom{5}{1}} \sim (NE)^{\binom{5}{2}} \sim (NE)^{\binom{5}{3}} \sim (NE)^{\binom{5}{4}} \sim E^{\binom{5}{5}}$$
$$\int_6 = N^{\binom{6}{0}} \sim (NE)^{\binom{6}{1}} \sim (NE)^{\binom{6}{2}} \sim (NE)^{\binom{6}{3}} \sim (NE)^{\binom{6}{4}} \sim (NE)^{\binom{6}{6}} \sim E^{\binom{6}{6}}$$
$$\int_7 = N^{\binom{7}{0}} \sim (NE)^{\binom{7}{1}} \sim (NE)^{\binom{7}{2}} \sim (NE)^{\binom{7}{3}} \sim (NE)^{\binom{7}{4}} \sim (NE)^{\binom{7}{5}} \sim (NE)^{\binom{7}{6}} \sim E^{\binom{7}{7}}$$
$$\int_8 = N^{\binom{8}{0}} \sim (NE)^{\binom{8}{1}} \sim (NE)^{\binom{8}{2}} \sim (NE)^{\binom{8}{3}} \sim (NE)^{\binom{8}{4}} \sim (NE)^{\binom{8}{5}} \sim (NE)^{\binom{8}{6}} \sim (NE)^{\binom{8}{7}} \sim E^{\binom{8}{8}}$$
$$\int_9 = N^{\binom{9}{0}} \sim (NE)^{\binom{9}{1}} \sim (NE)^{\binom{9}{2}} \sim (NE)^{\binom{9}{3}} \sim (NE)^{\binom{9}{4}} \sim (NE)^{\binom{9}{5}} \sim (NE)^{\binom{9}{6}} \sim (NE)^{\binom{9}{7}} \sim (NE)^{\binom{9}{8}} \sim E^{\binom{9}{9}}$$

Fig. 6: Sum of powers of cardinal points in combinatorial form

The combinatorial form are defined using the factorial and generates the coefficients of binomial expansion are presented in Fig. 7. The mathematical definition of the combinatorial form is given by:

$$\left(\frac{k}{i}\right) = \frac{k!}{(k-1)!_i!}, k \geq 0, i \geq 0$$

$$\int_0 = \binom{0}{0}$$
$$\int_1 = \binom{1}{0} \sim \binom{1}{1}$$
$$\int_2 = \binom{2}{0} \sim \binom{2}{1} \sim \binom{2}{2}$$
$$\int_3 = \binom{3}{0} \sim \binom{3}{1} \sim \binom{3}{2} \sim \binom{3}{3}$$
$$\int_4 = \binom{4}{0} \sim \binom{4}{1} \sim \binom{4}{2} \sim \binom{4}{3} \sim \binom{4}{4}$$
$$\int_5 = \binom{5}{0} \sim \binom{5}{1} \sim \binom{5}{2} \sim \binom{5}{3} \sim \binom{5}{4} \sim \binom{5}{5}$$
$$\int_6 = \binom{6}{0} \sim \binom{6}{1} \sim \binom{5}{2} \sim \binom{6}{3} \sim \binom{6}{4} \sim \binom{6}{5} \sim \binom{6}{6}$$
$$\int_7 = \binom{7}{0} \sim \binom{7}{1} \sim \binom{7}{2} \sim \binom{7}{3} \sim \binom{7}{4} \sim \binom{7}{5} \sim \binom{7}{6} \sim \binom{7}{7}$$
$$\int_8 = \binom{8}{0} \sim \binom{8}{1} \sim \binom{8}{2} \sim \binom{8}{3} \sim \binom{8}{4} \sim \binom{8}{5} \sim \binom{8}{6} \sim \binom{8}{7} \sim \binom{8}{8}$$
$$\int_9 = \binom{9}{0} \sim \binom{9}{1} \sim \binom{9}{2} \sim \binom{9}{3} \sim \binom{9}{4} \sim \binom{9}{5} \sim \binom{9}{6} \sim \binom{9}{7} \sim \binom{9}{8} \sim \binom{9}{9}$$

Fig. 7: Powers of cardinal as combinatorial form

2.2.1 Lemma

The sum of powers of the cardinal points of the intermediate cardinal points is equal to the values of the coefficient of binomial expansion that are the entries of Pascal triangle. These entries are defined by the combinatorial function.

$${}^nC_r = \binom{n}{r} = x + y = P_n^{x+y} : n \geq 0, r \leq n.$$

$$\Rightarrow {}^nC_r = N_{ij}^x . E_{ij}^y = f_{ij}(x, y) = (x + y)_{ij}$$

If N = path/point, East = path/point

then assume that

Path ≡ North ≡ Point ≡ East ≡ P

$$\Rightarrow N_{ij}^{x}.\,E_{ij}^{y} = P_{ij}^{x}.\,P_{ij}^{y} = P_{ij}^{x+y} = [x+y]_{ij}$$

$$\Rightarrow {}^{n}C_{r} = \binom{n}{r} = P_{n}^{x+y} = P_{ij}^{x+y} = [x+y]_{ij}$$

$$\Rightarrow P_{n}^{[x+y]_{r}} = {}^{n}C_{r} = \binom{n}{r} = [x+y]_{n}^{r} \text{ or } [x+y]_{r}^{n}$$

$$= {}^{n}C_{r} = N^{x}E^{y}$$

$$\int_{n} = (N^{x}E^{y}) = {}^{n}C_{r} = \binom{n}{r} = [x+y]_{(n,r)} : r \le n \ge 0$$

The Fig. 8 shows a relation in which the values of the combinatorial coefficients are exactly equal to the entries of the Pascal triangle. Elements in the rows are obtained by finding the sum of the two closet numbers above it in the succeeding row.

$$\int_{n} = \binom{n}{r}, n \ge 0, r \le n.$$

$$\int_{0} = \binom{0}{0} = 1, : n = r = 0$$

$$\int_{1} = \binom{1}{0} \sim \binom{1}{1} \Rightarrow 1 \sim 1 : n = 1, r = 0, 1.$$

$$\int_{2} = \binom{2}{0} \sim \binom{2}{1} \sim \binom{2}{2} \Rightarrow 1 \sim 2 \sim 1 : n = 2, r = 0, 1, 2.$$

$$\int_{3} = \binom{3}{0} \sim \binom{3}{1} \sim \binom{3}{2} \sim \binom{3}{3} \Rightarrow 1 \sim 3 \sim 3 \sim 1 : n = 3, r = 0, 1, 2, 3.$$

$$\int_{4} = \binom{4}{0} \sim \binom{4}{1} \sim \binom{4}{2} \sim \binom{4}{3} \sim \binom{4}{4} \Rightarrow 1 \sim 4 \sim 6 \sim 4 \sim 1 : n = 4, r = 0, 1, 2, 3, 4.$$

$$\int_{5} = \binom{5}{0} \sim \binom{5}{1} \sim \binom{5}{2} \sim \binom{5}{3} \sim \binom{5}{4} \sim \binom{5}{5}$$

$$\Rightarrow 1 \sim 5 \sim 10 \sim 10 \sim 5 \sim 1 : n = 5, r = 0, 1, 2, 3, 4, 5.$$

$$\int_{6} = \binom{6}{0} \sim \binom{6}{1} \sim \binom{5}{2} \sim \binom{6}{3} \sim \binom{6}{4} \sim \binom{6}{5} \sim \binom{6}{6}$$

$$\Rightarrow 1 \sim 6 \sim 15 \sim 20 \sim 15 \sim 6 \sim 1 : n = 6, r = 0, 1, 2, 3, 4, 5, 6.$$

$$\int_{7} = \binom{7}{0} \sim \binom{7}{1} \sim \binom{7}{2} \sim \binom{7}{3} \sim \binom{7}{4} \sim \binom{7}{5} \sim \binom{7}{6} \sim \binom{7}{7}$$

$$\Rightarrow 1 \sim 7 \sim 21 \sim 35 \sim 35 \sim 21 \sim 7 \sim 1 : n = 7, r = 0, 1, 2, 3, 4, 5, 6, 7.$$

$$\int_{8} = \binom{8}{0} \sim \binom{8}{1} \sim \binom{8}{2} \sim \binom{8}{3} \sim \binom{8}{4} \sim \binom{8}{5} \sim \binom{8}{6} \sim \binom{8}{7} \sim \binom{8}{8}$$

$$\Rightarrow 1 \sim 8 \sim 28 \sim 56 \sim 70 \sim 56 \sim 28 \sim 8 \sim 1 : n = 8, r = 0, 1, 2, 3, 4, 5, 6, 7, 8.$$

$$\int_{9} = \binom{9}{0} \sim \binom{9}{1} \sim \binom{9}{2} \sim \binom{9}{3} \sim \binom{9}{4} \sim \binom{9}{5} \sim \binom{9}{6} \sim \binom{9}{7} \sim \binom{9}{8} \sim \binom{9}{9}$$

$$\Rightarrow 1 \sim 9 \sim 36 \sim 84 \sim 126 \sim 126 \sim 84 \sim 36 \sim 9 \sim 1 : n = 9, r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.$$

Fig. 8: Triddle

$$\int_{n} = \binom{n}{r}, r \in [0, n] \; >_{<} + = {}^{n}C_{r}$$

2.2.2 Comparism

The combinatorial and sum of powers of cardinal coordinates generates Pascal entries. The relation in the Fig. 9 shows the number of rows under consideration with combinatorial and cardinal coordinates powers with respect to their values.

$$\int_{0} = \binom{0}{0} \equiv N^{1}E^{0} = (1,0) \Rightarrow 1 + 0 = 1$$

$$\int_{1} = \begin{cases} \binom{1}{0} \equiv N^{1}E^{0} = (1,0) \Rightarrow 1 + 0 = 1 \\ \binom{1}{1} \equiv N^{0}E^{1} = (0,1) \Rightarrow 0 + 1 = 1 \end{cases}$$

$$\int_{2} = \begin{cases} \binom{2}{0} \equiv N^{1}E^{0} = (1,0) \Rightarrow 1 + 0 = 1 \\ \binom{2}{1} \equiv N^{1}E^{1} = (1,1) \Rightarrow 1 + 1 = 2 \\ \binom{2}{2} \equiv N^{0}E^{1} = (0,1) \Rightarrow 0 + 1 = 1 \end{cases}$$

$$\int_{3} = \begin{cases} \binom{3}{0} \equiv N^{1}E^{0} = (1,0) \Rightarrow 1 + 0 = 1 \\ \binom{3}{1} \equiv N^{2}E^{1} = (2,1) \Rightarrow 1 + 1 = 3 \\ \binom{3}{2} \equiv N^{1}E^{2} = (1,2) \Rightarrow 0 + 1 = 3 \\ \binom{3}{3} \equiv N^{0}E^{1} = (0,1) \Rightarrow 0 + 1 = 1 \end{cases}$$

$$\int_{4} = \begin{cases} \binom{4}{0} \equiv N^{1}E^{0} = (1,0) \Rightarrow 1 + 0 = 1 \\ \binom{4}{1} \equiv N^{3}E^{1} = (3,1) \Rightarrow 3 + 1 = 4 \\ \binom{4}{2} \equiv N^{3}E^{3} = (3,3) \Rightarrow 3 + 3 = 6 \\ \binom{4}{3} \equiv N^{1}E^{3} = (1,3) \Rightarrow 1 + 3 = 4 \\ \binom{4}{4} \equiv N^{0}E^{1} = (0,1) \Rightarrow 0 + 1 = 1 \end{cases}$$

$$\int_{5} = \begin{cases} \binom{5}{0} \equiv N^{1}E^{0} = (1,0) \Rightarrow 1 + 0 = 1 \\ \binom{5}{1} \equiv N^{4}E^{1} = (4,1) \Rightarrow 4 + 1 = 5 \\ \binom{5}{2} \equiv N^{6}E^{4} = (6,4) \Rightarrow 6 + 4 = 10 \\ \binom{5}{3} \equiv N^{4}E^{6} = (4,6) \Rightarrow 4 + 6 = 10 \\ \binom{5}{4} \equiv N^{1}E^{4} = (1,4) \Rightarrow 1 + 4 = 5 \\ \binom{5}{5} \equiv N^{0}E^{1} = (0,1) \Rightarrow 0 + 1 = 1 \end{cases}$$

$$\int_{6} = \begin{cases} \binom{6}{0} \equiv N^{1}E^{0} = (1,0) \Rightarrow 1 + 0 = 1 \\ \binom{6}{1} \equiv N^{5}E^{1} = (5,1) \Rightarrow 5 + 1 = 6 \\ \binom{6}{2} \equiv N^{10}E^{5} = (10,5) \Rightarrow 10 + 5 = 15 \\ \binom{6}{3} \equiv N^{10}E^{10} = (10,10) \Rightarrow 10 + 10 = 20 \\ \binom{6}{4} \equiv N^{5}E^{10} = (5,10) \Rightarrow 5 + 10 = 15 \\ \binom{6}{5} \equiv N^{1}E^{5} = (1,5) \Rightarrow 1 + 5 = 6 \\ \binom{6}{6} \equiv N^{0}E^{1} = (0,1) \Rightarrow 0 + 1 = 1 \end{cases}$$

$$\int_7 = \begin{cases} \binom{7}{0} \equiv N^1E^0 = (1,0) \Rightarrow 1+0=1 \\ \binom{7}{1} \equiv N^6E^1 = (6,1) \Rightarrow 6+1=7 \\ \binom{7}{2} \equiv N^{15}E^6 = (15,6) \Rightarrow 15+6=21 \\ \binom{7}{3} \equiv N^{20}E^{15} = (20,15) \Rightarrow 20+15=35 \\ \binom{7}{4} \equiv N^{15}E^{20} = (15,20) \Rightarrow 15+20=35 \\ \binom{7}{5} \equiv N^6E^{15} = (6,15) \Rightarrow 6+15=21 \\ \binom{7}{6} \equiv N^1E^6 = (1,6) \Rightarrow 1+6=7 \\ \binom{7}{7} \equiv N^0E^1 = (0,1) \Rightarrow 0+1=1 \end{cases}$$

$$\int_8 = \begin{cases} \binom{8}{0} \equiv N^1E^0 = (1,0) \Rightarrow 1+0=1 \\ \binom{8}{1} \equiv N^7E^1 = (7,1) \Rightarrow 7+1=8 \\ \binom{8}{2} \equiv N^{21}E^7 = (21,7) \Rightarrow 21+7=28 \\ \binom{8}{3} \equiv N^{35}E^{21} = (35,21) \Rightarrow 35+21=56 \\ \binom{8}{4} \equiv N^{35}E^{35} = (35,35) \Rightarrow 35+35=70 \\ \binom{8}{5} \equiv N^{21}E^{35} = (21,35) \Rightarrow 21+35=56 \\ \binom{8}{6} \equiv N^7E^{21} = (7,21) \Rightarrow 7+21=28 \\ \binom{8}{7} \equiv N^1E^7 = (1,7) \Rightarrow 1+7=8 \\ \binom{8}{8} \equiv N^0E^1 = (0,1) \Rightarrow 0+1=1 \end{cases}$$

$$\int_9 = \begin{cases} \binom{9}{0} \equiv N^1E^0 = (1,0) \Rightarrow 1+0=1 \\ \binom{9}{1} \equiv N^8E^1 = (8,1) \Rightarrow 8+1=9 \\ \binom{9}{2} \equiv N^{28}E^8 = (28,8) \Rightarrow 28+8=36 \\ \binom{9}{3} \equiv N^{56}E^{28} = (56,28) \Rightarrow 56+28=84 \\ \binom{9}{4} \equiv N^{70}E^{56} = (70,56) \Rightarrow 70+56=126 \\ \binom{9}{5} \equiv N^{56}E^{70} = (56,70) \Rightarrow 56+70=126 \\ \binom{9}{6} \equiv N^{28}E^{56} = (28,56) \Rightarrow 28+56=84 \\ \binom{9}{7} \equiv N^8E^{28} = (8,28) \Rightarrow 8+28=36 \\ \binom{9}{8} \equiv N^1E^8 = (1,8) \Rightarrow 1+8=9 \\ \binom{9}{9} \equiv N^0E^1 = (0,1) \Rightarrow 0+1=1 \end{cases}$$

Fig 9: Moddle-triddle

2.2.3 Proposition 1

Limit as the higher powers of binomial expression (with respect to the pair of cardinals) tend to infinity/to a fixed point k, the angle of inclination between any two points tend to zero.

Since the combinatorial expression compared with the bases (N, E) raised to power (x, y) reveals a significant result. We need to explore the set of coefficients of binomial expansion to expose the variation of angle θ^0 at every disposition of points within the plane.

2.2.4 Proposition 2

$\not\to x, y \exists \int_n = P_n^{x+y} = x + y = 45^0 \Leftrightarrow x = y$

Fig. 10 is the representation of the binomial coefficient with respect to the degree of inclination of the direction, the point location in the first quadrant. The implication on polar coordinates. Application in graphing principle in which the fundamental Graphing principle for polar equations state that the graph of an equation in polar coordinates is the set of points which satisfy the equation that is, a point P (r, θ) is on the graph of an equation if and only if there is a representation of P, say (r, θ), such that r' and θ' satisfy the equation (OER Maths) [8].

Generally:

$\int_0 = \{\binom{0}{0}\} \equiv 0^0$

$\int_1 = \{\binom{1}{0}, \binom{1}{1}\} \equiv 90^0$

$\int_2 = \{\binom{2}{0}, \binom{2}{1}, \binom{2}{2}\} \equiv 45^0$

$\int_3 = \{\binom{3}{0}, \binom{3}{1}, \binom{3}{2}, \binom{3}{3}\} \equiv 22.5^0$

$\int_4 = \{\binom{4}{0}, \binom{4}{1}, \binom{4}{2}, \binom{4}{3}, \binom{4}{4}\} \equiv 11.25^0$

$\int_5 = \{\binom{5}{0}, \binom{5}{1}, \binom{5}{2}, \binom{5}{3}, \binom{5}{4}, \binom{5}{5}\} \equiv 5.625^0$

$\int_6 = \{\binom{6}{0}, \binom{6}{1}, \binom{5}{2}, \binom{6}{3}, \binom{6}{4}, \binom{6}{5}, \binom{6}{6}\} \equiv 2.8125^0$

$\int_7 = \{\binom{7}{0}, \binom{7}{1}, \binom{7}{2}, \binom{7}{3}, \binom{7}{4}, \binom{7}{5}, \binom{7}{6}, \binom{7}{7}\} \equiv 1.40625^0$

$\int_8 = \{\binom{8}{0}, \binom{8}{1}, \binom{8}{2}, \binom{8}{3}, \binom{8}{4}, \binom{8}{5}, \binom{8}{6}, \binom{8}{7}, \binom{8}{8}\} \equiv 0.703125^0$

$\int_9 = \{\binom{9}{0}, \binom{9}{1}, \binom{9}{2}, \binom{9}{3}, \binom{9}{4}, \binom{9}{5}, \binom{9}{6}, \binom{9}{7}, \binom{9}{8}, \binom{9}{9}\} \equiv 0.351562^0$

$\int_{10} = \{\binom{10}{0}, \binom{10}{1}, \binom{10}{2}, \binom{10}{3}, \ldots\ldots, \binom{10}{8}, \binom{10}{9}, \binom{10}{10}\} \equiv 0.175782125^0$

$\int_{11} = \{\binom{11}{0}, \binom{11}{1}, \binom{11}{2}, \binom{11}{3}, \ldots\ldots, \binom{11}{9}, \binom{11}{10}, \binom{11}{11}\} \equiv 0.08789062^0$

$\int_{12} = \{\binom{12}{0}, \binom{12}{1}, \binom{12}{2}, \binom{12}{3}, \ldots\ldots, \binom{12}{9}, \binom{12}{10}, \binom{12}{11}, \binom{12}{12}\} \equiv 0.04394531^0$

$\int_{13} = \{\binom{13}{0}, \binom{13}{1}, \binom{13}{2}, \binom{13}{3}, \ldots\ldots, \binom{13}{9}, \binom{13}{10}, \binom{13}{11}, \binom{13}{12}\}, \binom{13}{13} \equiv 0.02197656^0$

$\int_{14} = \{\binom{14}{0}, \binom{14}{1}, \binom{14}{2}, \binom{14}{3}, \ldots\ldots, \binom{14}{9}, \binom{14}{10}, \binom{14}{11}, \binom{14}{12}, \binom{14}{13}, \binom{14}{14}\} \equiv 0.010986328^0$

$\int_{15} = \{\binom{15}{0}, \binom{15}{1}, \binom{15}{2}, \binom{15}{3}, \ldots\ldots, \binom{15}{10}, \binom{15}{11}, \binom{15}{12}, \binom{15}{13}, \binom{15}{14}, \binom{15}{15}\} \equiv 0.005493164063^0$

Fig. 10: Angle between north and east ($0^0 \leq \theta^0 \leq 90^0$, $\theta^0 = 45^0 \Leftrightarrow x = y$)

Thus,

$\lim \int_n = \frac{\theta^0}{n}$, $\not\to n \geq 1, n \neq 0, \theta^0 \geq 0$

$n - k < \infty$

$\Rightarrow \quad \left|\int n = \frac{\theta^0}{n}\right| = \epsilon_n : \epsilon_n \geq 0.$

2.3 Further Observation

Let θ^0 be expressed approximately to two decimal places.

The Fig. 11 the values of the degrees, in the first quadrant. The vertical line and the horizontal line (North, East), is perpendicular at 90^0. Thus, the subdivisions indicate the order of the breakdown of the ordinal points in degrees.

$\int_n = {}^nC_r$

$$\begin{cases} 0.00^0 : n = r = 0 \\ 90.00^0 : n = 1, r = 0, 1. \\ 45.00^0 : n = 2, r = 0, 1, 2. \\ 22.50^0 : n = 3, r = 0, 1, 2, 3. \\ 11.25^0 : n = 4, r = 0, 1, 2, 3, 4 . \\ 5.62^0 : n = 5, r = 0, 1, 2, 3, 4, 5. \\ 2.81^0 : n = 6, r = 0, 1, 2, 3, 4, 5, 6. \\ 1.40^0 : n = 7, r = 0, 1, 2, 3, 4, 5, 6, 7. \\ 0.70^0 : n = 8, r = 0, 1, 2, 3, 4, 5, 6, 7, 8. \\ 0.35^0 : n = 9, r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. \\ 0.17^0 : n = 10, r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. \\ 0.08^0 : n = 11, r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. \\ 0.04^0 : n = 12, r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. \\ 0.02^0 : n = 13, r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13. \\ 0.01^0 : n = 14, r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14. \\ 0.00^0 : n = 15, r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9,10,11,12,13,14,15. \end{cases}$$

Fig. 11: Order of angles variation

That is the limit of θ^0.

The question at heart of the matter is, why does the limit exist at n=15? The idea behind 15 is that it was known that the number of groundnut bags that form the pyramid is equal to 15 x 10^m for some m>0 $\epsilon >_{<}+$.

The limit is 15.

$\Rightarrow \lim \int n = \frac{\theta^0}{15} = 0$ for some $\theta^0 \leq \theta^0 \leq 90^0$.

n → 15 $\Rightarrow \int n = \frac{\theta^0}{15} = \epsilon_{15} \geq 0.$

3.0 Results and Discussions

3.1 Theorem 1:

The sum of powers of cardinal coordinates is equal to the coefficients of binomial expansion (elements of Pascal triangle).

$\int_n$ **8=** $(N^x E^y) = P_n^x . P_n^y = P_n^{x+y} = [x + y]_n = {}^nC_r$

for all n ≥ 0, r ≤ n ⊀ r ϵ positive integers.

Thus, the combinatorial form of the binomial coefficients (Pascal triangle entries) is mapped to corresponding successive sum of powers of the pair of cardinal points on the plane. The proof is a direct proof (Modus ponens) by comparism of corresponding entries of cardinal coordinates in the plane. The implication of the theorem in this article is the extension of the ordinal direction applied on the compass and use for the location of points and determination of angles of inclination of the point. The angles tend to a limit as the powers of the ordinal direction increases. The angles between two successive points tend to zero as the cardinal powers tend to infinity. This implies the convergence of ordinal points. The convergence implies that the higher ordinal points above the limit of convergence are dense in a particular point on the plane. The angle between North and East is 90^0 tend to zero as the powers of the ordinal points increases. Thus the dense ordinal points are located at the point where the limit of the degree of inclination tend to zero.

That is:

$lim|\theta^0| \rightarrow 0$

$(x, y) \rightarrow \infty$

This implies $lim|\theta^0| \rightarrow 0 \; as \; (x,y) \rightarrow \infty$

where $\theta^0 = 90^0$

The limit as angle between North and East tend to zero, the powers of the pairs of coordinates of North and East tend to infinity.

4.0 Conclusions

This article established a clear relationship between the cardinals coordinates points and the coefficients of the binomial expression expansion. The sum of powers of the cardinal points generates corresponding elements of Pascal triangle with respect to Descarte Cartesian Plane. The results reveals that such sum of powers are compared with the combinatorial expression.

The implication of the results shows that the binomial theorem can be applied to space maths with bearings on the plane. Perhaps, by assumption, the critical thinking great minds (Mathematicians) Pascal and Bernoulli might have derived the ideas from the space maths analysis. As such, binomial theorem involves concepts of space mathematics (astronomy/astrology). The space is defined with respect to a well-defined cardinal regions based on an origin.

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Declaration of conflict of interest

The authors declare no conflict of interest.

References

Dan L. and Anders T. (1994). Mathematica Scandinavica vol 74, No. 1, pp.19-33. Counting matrices with co-ordinates infinite fields.

Hans W. (1975). Model of the Pascal Pyramid displaying the arrangement in the space of trinomial coefficient.

Lauret M (2017). 1060-Trigonometry Pilot Edition, Spring. Salt Lake Community College and University of Utah.

Meadows and Peter. (2013). Solar observing Parallactic Angle. November.

Pedro V. S., de Vilhena, (2003). Sum of Powers Applied in Cartessian Plane. www.icsrs.org. Available online at http://www.geman.in

Papri D., Paul G. and Nidhi K. (2020). Co-ordinate wise powers of algebraic Varieties.

Xin H., Yi H. and Jia L. (2006). Experience with the cardinal co-ordinate system contribute to the precision of cognitive maps. Burgess 2006. Carpenter et al., 2015.