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An Exposition of Integral Powers of Polynomial with Binomial Coefficients

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Abstract
This article exposed the disposition of powers of a polynomial function with binomial coefficients. The relationship was established by the deft deployment of summation notation, combinatorial techniques, and differential operators. The result of the investigation shows clearly that the disposition of integrals powers of polynomials with binomial coefficients generates even positive integer factorial values (2k)!.

Nomenclature and units

sh	Sunshine hours
rsh	Relative sunshine hours
ϕ	Latitude
ΔT	Change in temperature
T_{av}	Average temperature
H	Humidity
RH	Relative humidity

1.0 Introduction

A number which has a rational root is called a perfect power Chase, (1844). An integral perfect power is the irrational root that is an integer. In their contributions in Exponential Diophantine Equations, Shorey and Tijdeman (1988). investigated perfect powers at integral values of a polynomial powers at integral values of a polynomial and obtained the following result:

Let f (x) be a polynomial with rational integer coefficients and with at least two simple rational zeros.

Suppose b ≠ 0, m ≥ 0, *x* and y with /Y/ are rational integers. Then the equation:

$$f(x) = by^m$$

Implies that m is bounded by a computable number depending only on b and f (*x*).

There was a discussion on orders of difference of integral perfect powers by Ibrahim et al. The investigation reveals that if any number of consecutive integers are raised to a positive integral power k, then the k difference is equal to k!.

Based on their observation, it shows that for a given sequence of perfect powers [$f_j \leq j$], an examination of the set of perfect squares {0, 1, 4, 9, 16, 25, 36, 49} reveals that the difference between two successive elements is odd. The actual set of such differences is {1, 3, 5, 7, 9, 11, 13}. Furthermore, the set of differences of the preceding difference set is constant which is {2, 2, 2, 2, 2, 2} = {2}, a singleton set. Clearly the set of difference of the above second difference set is {0}. They explore the emerging pattern in their investigation and found out that this pattern have persist for all perfect squares resulting from the set of integers. The result of their preliminary theorem says that if $f_j = j$ for integral j.

Then

(i) Δf_j^2 is odd

(ii) $\Delta^2(f_j^2) = 2$

(iii) $\Delta^k(f_j^2) = 0, k \in \{3, 4 \ldots\}$

A polynomial function is a very important function applied in some real life problem. A polynomial function is defined mathematically below:

Let n be a non-negative integer and let $\propto_n, \propto_{n-1}, \ldots, \propto_2, \propto_1, \propto_0$, be real numbers with $\propto n \neq 0$. The function defined by

$$f(j) = \sum_{n=0}^{n} \propto_{n} j^n$$

$$f(j) = \propto_n j^n + \propto_{n-1} j^{n-1} + \ldots + \propto_2 j^2 + \propto_1 j + \propto_0$$

is called a polynomial function of *x* of degree n. The number $\propto_n$, the coefficient of the variable j^n to the highest power, is called the leading highest coefficient, which determine the pivotal element of the polynomial function. A polynomial function is a differentiable function. A polynomial function is determined with respect to the highest power of the variable of the function. A relationship between the power and coefficient of a polynomial was examined.

In this article, an investigation into the powers of polynomial functions whose coefficients are in form of combinatorial function.

The examination on the polynomial function:

$$fj = j^2 + j$$

$$f_j^i = 2j^{2-1} + j^{1-1}$$

$$f_j^i = 2j + j^0$$

$$fj = 2j + 1$$

$$f_j^i = \text{odd for all } j \geq 0$$

$$f_j^{ii} = 2j^{1-1} + 0$$

$$f_j^{ii} = 2$$

The above results shows that the second derivative of the function in equation I is equal to 2 = 2! = $[2\ (1)]!$

It is observed that the coefficients of the polynomial terms is equal to one, which is equivalent to the coefficient of binomial coefficient if the power is equal to 1 and can be expressed in combinatorial form.

Defined factional function as: ${}^n_rC = \frac{n!}{(n-r)!r!}$

which implies that 1 = $\binom{1}{0}$ ………………………………………… (1)

$$= \frac{1!}{(1-0)!0!}$$

$$= \frac{1}{1!\ 0!}$$

$$= \frac{1}{1x1}$$

$$= 1$$

Similarly, the coefficient of j^1 is 1.

This is also expressed as:

$$1 = \binom{1}{1}$$

$$= \frac{1!}{(1-1)!1!}$$

$$= \frac{1!}{0!\ 1!}$$

$$= \frac{1}{1x1}$$

= 1

We have the coefficient of j^2 is 1 and

$$1 = \binom{1}{0} \quad \ldots\ldots(2)$$

and the coefficient of j^1 is 1. Similarly,

$$1 = \binom{1}{1} \quad \ldots\ldots(3)$$

Clearly, the coefficients are binomial coefficient which are entries of the Pascal triangle. The relation between the power of the last term of the polynomial (1) and the power of the first term is that: the lowest power multiply by two will yield the highest power of the polynomial function. That is, "if the lowest power is 1 from equation (I) then the highest power is 2(1).

The emerging pattern motives the investigation of whether or not this pattern will persist for higher powers of such polynomials with binomial coefficients. For more definitions and elementary results on perfect powers, (Hill 2005; Andreescu (2009); Perfect Powers (2015).

This paper tries to investigate the expositional disposition of integral power of binomial in a polynomial, with respect to the binomial coefficients.

Review of literature shows that no such investigation has been undertaken. Thus this article adds to the existing body of knowledge, by providing more details on binomial coefficient of polynomial.

2.0 Materials and Methods

2.1 Preliminary Definitions

In what follows, differential operators will be defined.

The differential operator is an operator that differentiate a function. In the process of differentiation, the operator operates with respect to the powers and coefficients of the variable of the terms in a given function.

The power of each term is multiplied by the coefficient of the term and the power is reduced by one to express the derivative of the term. This process is applied to each term of the function. This is expressed mathematically below.

2.2 Differential Operator

Given a polynomial function

$$(fj) = \sum_{i=0}^{n} \propto_i j^i \ldots\ldots(4)$$

Define the derivative on the polynomial function as:

$$(f_j^1) = \frac{d}{dj}\left[\propto_i j^i\right] \ldots\ldots(5)$$

$$= \propto_i \frac{d}{dj}\left[j^i\right]$$

$$= \propto_i . i . j^{i-1}$$

$$= \propto_i i\, j^{i-1}$$

For every integral j, i ≥ 1

The implication of the differential operator is the derivative is determined by the operations on the power and coefficients of the polynomial term. It shows the importance of the coefficients and powers of a term of a function. It is clearly seen that the derivative of a term in a function is the product of the power of the term and its coefficient and raising the term a power which is less by one of the original power of the term.

3.0 Results

3.1 Preliminary Theorem:

Suppose that $(f_j) = \sum_{i=1} \propto_i j^i$

where, $\propto_i = \sum_{i=0}^{k} \binom{k}{i}$ for all values of k

$$\Rightarrow (f_j) = \begin{cases} \sum_{i=k}^{2k} j^i, & \text{polynomial power of terms} \\ \sum_{i=0}^{k} \binom{k}{i}, & \text{polynomial coefficient} \end{cases}$$

By relating the coefficients and polynomial terms, we have:

$$\sum \propto_i j^i = \sum_{i=0}^{k} \binom{k}{i} \left(\sum_{i=k}^{2k} j^i\right) \ldots\ldots(6)$$

$$= \sum_{i=0}^{k} \sum_{i=k}^{2k} \binom{k}{i} j^i \qquad \text{for } \begin{matrix} i \geq 0 \\ k \geq 0 \end{matrix}$$

We have that;

i. $f_{(j)} = j^{2k} + j^k \qquad k = 1$

ii. $f^k{}_{(j)} = 2kj^{2k-k} + kj^{k-k} = 2kj^k + k$

iii. $f^{2k}{}_{(j)} = 2kj^{k-k} = [2k]! = 2!$

3.1.1 Main Theorem.

In the sequel, we examine the computational disposition of

$$f_{(j)} = \sum \propto_i j^i = \sum_{i=0}^{k} \sum_{i=k}^{2k} \binom{k}{i} j^i$$

for k ≥ 2.

For k = 2,

$$f_{(j)} = \sum_{i=0}^{2} \sum_{i=2}^{4} \binom{2}{i} j^i$$

$$= j^4 + 2j^3 + j^2$$

The 4th derivative of $f_{(j)}$ is obtained by finding the 1st, 2nd, 3rd and then the 4th derivative, which is done as follows:

$$f_j = j^4 + 2j^3 + j^2$$

$$f_j^1 = \frac{d}{dj}[j^4 + 2j^3 + j^2]$$

The 1st derivative is:

$$f_{(j)}^{i} = \frac{d}{dj}(j^4) + 2\frac{d}{dj}(j^3) + \frac{d}{dj}(j^2)$$

$= 4j^{4-1} + 6j^{3-1} + 2j^{2-1}$

$= 4j^3 + 6j^2 + 2j^1$

The 2nd derivative is:

$$f_{(j)}^{ii} = \frac{d}{dj}[4j^3 + 6j^2 + 2j^1]$$

$$= 4\frac{d}{dj}(j^3) + 6\frac{d}{dj}(j^2) + 2\frac{d}{dj}j^1$$

$= 12j^{3-1} + 12j^{2-1} + 2j^{1-1}$

$= 12j^2 + 12j^1 + 2$

The 3rd derivative is:

$$f_{(j)}^{iii} = \frac{d}{dj}[12j^2 + 12j^1 + 2]$$

$$= 12\frac{d}{dj}(j^2) + 12\frac{d}{dj}(j^1) + \frac{d}{dj}(2)$$

$= 24j^{2-1} + 12j^{2-1} + 0$

$= 24j + 12$

The 4th derivative is:

$$f_{(j)}^{iv} = \frac{d}{dj}[24j + 12]$$

$$= 24\frac{d}{dj}(j) + \frac{d}{dj}(24)$$

$= 24j^{1-1} + 0$

= 24

= 4!

= [2(2)]!

= 2k! for k =2

For k = 3

$$f_{(j)} = \sum_{i=0}^{3} \sum_{i=3}^{6} \binom{3}{i} j^i$$

$$f_{(j)} = j^4 + 2j^3 + j^2$$

To obtain the 6th derivative, we follow a similar procedure as obtained in the 4th derivative.

The first derivative is:

$$f_{(j)}^{i} = \frac{d}{dj}[j^6 + 3j^5 + 3j^4 + j^3]$$

$= \frac{d}{dj}(j^6) + 3\frac{d}{dj}(j^5) + 3\frac{d}{dj}(j^4) + \frac{d}{dj}(j^3)$

$= 6j^{6-1} + 15j^{5-1} + 12j^{4-1} + 3j^{3-1}$

$= 6j^5 + 15j^4 + 12j^3 + 3j^2$

The second derivative is:

$$f_{(j)}^{ii} = \frac{d}{dj}[6j^5 + 15j^4 + 12j^3 + 3j^2]$$

$= 6\frac{d}{dj}(j^5) + 15\frac{d}{dj}(j^4) + 12\frac{d}{dj}(j^3) + 3\frac{d}{dj}(j^2)$

$= 30j^{5-1} + 60j^{4-1} + 36j^{3-1} + 6j^{2-1}$

$= 30j^4 + 60j^3 + 36j^2 + 6j^1$

The third derivative is:

$$f_{(j)}^{iii} = \frac{d}{dj}[30j^4 + 60j^3 + 36j^2 + 6j^1]$$

$= 30\frac{d}{dj}(j^4) + 60\frac{d}{dj}(j^3) + 36\frac{d}{dj}(j^2) + 6\frac{d}{dj}(j^1)$

$= 120j^{4-1} + 18j^{3-1} + 72j^{2-1} + 6$

$= 120j^3 + 180j^2 + 72j^1 + 0$

$= 120j^3 + 180j^2 + 72j^1$

The fourth derivative is:

$$f_{(j)}^{iv} = \frac{d}{dj}\left[120j^3 + 180j^2 + 72j\right]$$

$$= 120\frac{d}{dj}(j^3) + 180\frac{d}{dj}(j^2) + 72\frac{d}{dj}(j)$$

$$= 360j^{3-1} + 18j^{2-1} + 72$$

$$= 360j^2 + 360j^2 + 72$$

The fifth derivative is:

$$f_{(j)}^{v} = \frac{d}{dj}\left[360j^2 + 360j^3 + 72\right]$$

$$= 360\frac{d}{dj}(j^2) + 360\frac{d}{dj}(j) + \frac{d}{dj}(72)$$

$$= 720j^{2-1} + 360j^{1-1} + 0$$

$$= 720j^1 + 360$$

The sixth derivative is:

$$f_{(j)}^{vi} = \frac{d}{dj}\left[720j + 360\right]$$

$$= 720\frac{d}{dj}(j) + \frac{d}{dj}(360)$$

$$= 720j^{1-1} + 0$$

$$= 720$$

$$= [2(3)]!$$

$$= [2(k)]!$$

Thus by induction hypothesis, the validity of the polynomial at k = 1, implies the validity at k = 2. If it is valid for k = 1, 2, 3

Then it is valid for k ≥ 3.

For some ≥ 3

$$f^{2k}_{(j)} = \frac{d^{2k}}{dj^{2k}}\left[\sum_{i=0}^{k}\sum_{i=k}^{2k}\binom{k}{i}j^i\right] = [2(k)]! \text{ for } k \geq 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

4.0 Discussions

The pattern recognition involves the application of even factorial which can be generated through a polynomial function with respect to the coefficients of Pascal's triangle entries in a triangular fashion. The powers of the variable of the polynomial functions are distributed with respect to even and odd order of positive integers.

Yields:

$$j^2 + j$$

$$j^2 + j$$
$$j^4 + 2j^3 + j^2$$
$$j^6 + 3j^5 + 3j^4 + j^3$$
$$j^8 + 4j^7 + 6j^6 + 4j^5 + j^4$$
$$j^{10} + 5j^9 + 10j^8 + 10j^7 + 5j^6 + j^5$$
$$j^{12} + 6j^{11} + 15j^{10} + 20j^9 + 15j^8 + 6j^7 + j^6$$
$$j^{14} + 7j^{13} + 21j^{12} + 35j^{11} + 35j^{10} + 21j^9 + 7j^8 + j^7$$
$$j^{16} + 8j^{15} + 24j^{14} + 56j^{13} + 70j^{12} + 56j^{11} + 28j^{10} + 8j^9 + j^8$$

Fig. I: Triangular Form Pattern of Polynomials with Binomial Coefficients (Pascal entries).
for k ≥ 1.

1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

Fig. II: Pascal's tries Triangular form with respect to coefficients of Polynomial Function

2 1
4 3 2
6 5 4 3
8 7 6 5 4
10 9 8 7 6 5
12 11 10 9 8 7 6
14 13 12 11 10 9 8 7
16 15 14 13 12 11 10 9 8

Fig. III: Corresponding Pattern of Powers of Polynomials

Observed that the relation on the first and last term of polynomial in ascending or descending order implies that:

For all k = least power

There exist 2k = highest power.

$$\therefore \text{Power} = \begin{cases} k & \text{if least} \\ 2k & \text{if highest} \end{cases}$$

Defining the entries of the triangular form in Fig II: (Pascal triangle) with respect to the laws of indices.

Consider the product rule: $x^a . x^b = x^{a+b}$

Let x^1, x^1 be the first entries in the first row

Let $R_1 = row\ one$

Where the powers of the variable *x* are the coefficients of the polynomial function.

The power of first and least term is fixed at one (1).

This implies that:

$R_1 \quad = x^1, x^1$

$R_2 \quad = x^1, x^1 . x^1, x^1$

$= x^1, x^{1+1}, x^1$

$= x^1, x^2, x^1$

$R_3 \quad = x^1, x^1 . x^2, x^2 . x^1, x^1$

$= x^1, x^{1+2}, x^{2+1}, x^1$

$= x^1, x^3, x^3, x^1$

$R_4 \quad = x^1, x^1 . x^3, x^3 . x^3, x^3 . x^1, x^1$

$= x^1, x^{1+3}, x^{3+3}, x^{3+1}, x^1$

$= x^1, x^4, x^6, x^4, x^1$

$R_5 \quad = x^1, x^1 . x^4, x^4 . x^6, x^6 . x^4, x^4 . x^1, x^1$

$= x^1, x^{1+4}, x^{4+6}, x^{6+4}, x^{4+1}, x^1$

$= x^1, x^5, x^{10}, x^{10}, x^5, x^1$

$R_6 \quad = x^1, x^1 . x^5, x^5 x^{10}, x^{10} . x^{10}, x^{10} . x^5, x^5 . x^1 x^1$

$= x^1, x^{1+5}, x^{5+10}, x^{10+10}, x^{10+5}, x^{5+1}, x^1$

$= x^1, x^6, x^{15}, x^{20}, x^{15}, x^6, x^5$

$R_7 \quad = x^1, x^1 . x^6, x^6 x^{15}, x^{15} . x^{20}, x^{20} . x^{15}, x^{15} . x^6, x^6 . x^1, x^1$

$= x^1, x^{1+6}, x^{6+15}, x^{15+20}, x^{20+15}, x^{15+6}, x^{6+1}, x^1$

$= x^1, x^7, x^{21}, x^{35}, x^{35}, x^{21}, x^7, x^1$

$R_8 \quad = x^1, x^1 . x^7, x^7 . x^{21}, x^{21} . x^{35}, x^{35} . x^{35}, x^{35} . x^{21}, x^{21} . x^7, x^7 . x^1, x^1$

$= x^1, x^{1+7}, x^{7+21}, x^{21+35}, x^{35+35}, x^{35+21}, x^{21+7}, x^{7+1}, x^1$

$= x^1, x^8, x^{28}, x^{56}, x^{70}, x^{56}, x^{28}, x^8, x^1$

This yields:

$R_1 = x^1, x^1$

$R_2 = x^1, x^2, x^1$

$R_3 = x^1, x^3, x^3, x^1$

$R_4 = x^1, x^4, x^6, x^4, x^1$

$R_5 = x^1, x^5, x^{10}, x^{10}, x^5, x^1$

$R_6 = x^1, x^6, x^{15}, x^{20}, x^{15}, x^6, x^1$

$R_7 = x^1, x^7, x^{21}, x^{35}, x^{35}, x^{21}, x^7, x^1$

$R_8 = x^1, x^8, x^{28}, x^{56}, x^{70}, x^{56}, x^{28}, x^8, x^1$

The operation of multiplication and division of indices will be employed to define the entries of the triangular form of powers of the polynomial in Fig. III.

If 2k is the power of the first term, then k is the power of the last term.

For k ≥ 1

Let x^{2k}, x^k be the entries in the first row, were the powers of the variable x are the powers of the polynomial.

Consider the rule: $x^a . x^b = x^{a-b}$ (product)

$x^a \div x^b = x^{a-b}$ (quotient)

Multiply x^2 to the first term and x^1 to the last term in each row ≥ 2.

$R_1 = x^2, x^1$

$R_2 = x^2 . x^2, \frac{x^2 . x^1}{x^0}, x^1 . x^1$

$= x^{2+2}, x^{3-0}, x^2$

$= x^4, x^3, x^2$

$R_3 = x^4.x^2, \frac{x^4.x^3}{x^2}, \frac{x^3.x^2}{x^1}, x^2.x^1$

$= x^{4+2}, x^{7-2}, x^{5-1}, x^{2+1}$

$= x^6, x^5, x^4, x^3$

$R_4 = x^6.x^2, \frac{x^6.x^5}{x^4}, \frac{x^5.x^4}{x^3}, \frac{x^4.x^3}{x^2}, x^3.x^1$

$= x^{6+2}, x^{11-4}, x^{9-3}, x^{7-2}, x^{3+1}$

$= x^8, x^7, x^6, x^5, x^4$

$R_5 = x^8.x^2, \frac{x^8.x^7}{x^6}, \frac{x^7.x^6}{x^5}, \frac{x^6.x^5}{x^4}, \frac{x^5.x^4}{x^3}, x^4.x^1$

$= x^{8+2}, x^{15-6}, x^{13-5}, x^{11-4}, x^{9-3}, x^5$

$= x^{10}, x^9, x^8, x^7, x^6, x^5$

$R_6 = x^{10}.x^2, \frac{x^{10}.x^9}{x^8}, \frac{x^9.x^8}{x^7}, \frac{x^8.x^7}{x^6}, \frac{x^7.x^6}{x^5}, \frac{x^6.x^5}{x^4}, x^5.x^1$

$= x^{10+2}, x^{19-8}, x^{17-7}, x^{15-6}, x^{13-5}, x^{11-4}, x^{5+1}$

$= x^{12}, x^{11}, x^{10}, x^9, x^8, x^7, x^6$

$R_7 = x^{12}.x^2, \frac{x^{12}.x^{11}}{x^{10}},$

$\frac{x^{11}.x^{10}}{x^9}, \frac{x^{10}.x^9}{x^8}, \frac{x^9.x^8}{x^7}, \frac{x^8.x^7}{x^6}, \frac{x^7.x^6}{x^5}, x^6.x^1$

$= x^{12+2}, x^{23-10},$

$x^{21-9}, x^{19-8}, x^{17-7}, x^{15-6}, x^{13-5}, x^7$

$= x^{14}, x^{13}, x^{12}, x^{11}, x^{10}, x^9, x^8, x^7$

$R_8 = x^{14}.x^2, \frac{x^{14}.x^{13}}{x^{12}},$

$\frac{x^{13}.x^{12}}{x^{11}}, \frac{x^{12}.x^{11}}{x^{10}}, \frac{x^{11}.x^{10}}{x^9}, \frac{x^{10}.x^9}{x^8}, \frac{x^9.x^8}{x^7}, \frac{x^8.x^7}{x^6}, x^7.x^1$

$= x^{14+2}, x^{27-12},$

$x^{25-11}, x^{23-10}, x^{21-9}, x^{19-8}, x^{17-7}, x^{15-6}, x^{7+1}$

$= x^{16}, x^{15}, x^{14}, x^{13}, x^{12}, x^{11}, x^9, x^8$

This yields:

$R_1 = x^2, x^1$

$R_2 = x^4, x^3, x^2$

$R_3 = x^6, x^5, x^4, x^3$

$R_4 = x^8, x^7, x^6, x^5, x^4$

$R_5 = x^{10}, x^9, x^8, x^7, x^6, x^5$

$R_6 = x^{12}, x^{11}, x^{10}, x^9, x^8, x^7, x^6$

$R_7 = x^{14}, x^{13}, x^{12}, x^{11}, x^{10}, x^9, x^8, x^7$

$R_8 = x^{16}, x^{15}, x^{14}, x^{13}, x^{12}, x^{11}, x^{10}, x^9, x^8$

Finally, the relationship between the powers and binomial coefficients on the polynomial function is established. It has been clearly proven that the even factorial is generated from the polynomial function with respect to the differential operator.

Hence:

$$f^{2k}{}_{(j)} = [2(k)]!, \quad k \geq 1$$

Where:

$$f_{(j)} = \sum_{i=0}^{k} \sum_{i=k}^{2k} \binom{k}{i} j, \quad k \geq 1$$

5.0 Conclusions

This article exposed the structure of polynomial with respect to power and coefficient function. In particular, the implication of the powers is the pivotal element, which is the highest power in the polynomial function. Consequently, the pivotal element determines the order of the derivative of higher order. The differential operator was employ to prove the theorem on the even factorial generated with respect to the polynomial function. The order of the set of powers in the polynomial function is in the order of natural numbers. Based on the result of the theorem and that of Ladan et al. (2016) a comparison was made on the coefficient of the factorial and significant gab was found. The limitation in Ladan et al. (2016) was that the coefficient of $k!$ is equal to 1. In this work, it was found that the coefficient of $k!$ is equal to 2. A comparison of the results showed the variation in the value of the coefficient of $k!$ which equals $\{1,2\}$. The binomial theorem was known in Eastern cultures prior to its discovery in Europe. The same Mathematics is often discovered or invented by independent researchers separated by time, place and culture. Finally, we have seen clearly that the polynomials whose powers are defined by the summation notation.

$$\text{Power of Polynomials} = \sum_{i=k}^{2k} j^i, \quad k \geq 1$$

$$\text{Coefficient of Polynomials} = \sum_{i=0}^{k} \binom{k}{i}, \quad k \geq 1$$

$$\text{Polynomial Polynomials} = \sum_{i=0}^{k} \sum_{i=k}^{2k} \binom{k}{i} j^{i}, \quad k \geq 1$$

Thus

$$f^{2k}{}_{(j)} = \frac{d^{2k}}{dj^{2k}} \left[\sum_{i=0}^{k} \sum_{i=k}^{2k} \binom{k}{i} j^{i} \right] = [2(k)]!$$

$$= [2(k)]!, \ \text{for}\, k \geq 1$$

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The authors declare no conflict of interest.

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